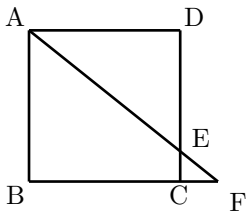
Clover Math Competition

Form B

April 26, 2024

1. Find the sum of the first 5 odd numbers.
2. Suppose that William is 4ft 3in and his brother is 5ft 2in. How many inches taller is William's brother than William, given that a foot is equivalent to 12 inches?
3. Find the number of positive integers less than or equal to 20 that are divisible by 2 but not by 6.
4. Brain Bean is picking flowers in a field of 35 clovers. Three in every five clovers will always bloom a flower. If he picks the entire field, how many flowers will she have?
5. Bunny rolls a fair 6-sided dice. What is the probability the number she rolls is less than 4?
6. Three mathematicians did three math problems in three minutes. How many math problems will two mathematicians do in six minutes?
7. If the 4 corners of a square are folded inwards so that they touch the center of the square, what is the ratio of the area of the original figure to the new figure?
8. The number e, also known as Euler's number, is approximately equal to 2.718. Which integer is closest to $5e$?
9. Suppose that we define a new function \$ such that $a\$b = ab - \frac{a}{b}$. What is 5\$3, expressed as a common fraction?
10. Arthur is 25% taller than Brian and Andrew is 20% taller than Arthur. What percent taller is Andrew than Brian?
11. If $12x + 12y = 30$, what is $20x + 20y$?
12. Square $ABCD$ has side length of 20. The length of DE is four times as long as the length of EC. Find the area of triangle CEF.



13. A math competition has two contests. One is 15 minutes long and the other is 36 minutes long. Each begins immediately after the previous one ends. Both competitions start at 9 A.M. and run continuously until 2 P.M., such that as soon as a contest ends, another round of it begins. When will the competitions start at the same time again?
14. A box has width 4, length 5, and height 6. The top face of the box is open and the bottom of the box has a hole cut into it whose side lengths are half the side lengths of the base. What is the inner surface area of the box?
15. You have two gallons of a solution that is 75% orange juice and 25% water. However, you want a solution that is half orange juice and half water. How many gallons of pure water should you add?

16. The sum of 3 consecutive integers is 777. What is the second largest of the three integers?

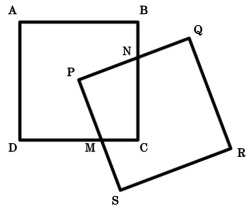
17. There are 7 people at a dinner table, all of whom want to eat strawberries. When they split the strawberries equally among themselves, there are 3 leftover. Suddenly, Ron and Hermione have to leave the party. When the remaining people redistributed the strawberries equally among themselves, there were still 3 leftovers. Find the smallest number of strawberries (larger than 3) that could have been at the table.

18. Find n such that the nth term in this sequence is equivalent to $\frac{1}{2}$: $\frac{40}{15}, \frac{41}{18}, \frac{42}{21}, \frac{43}{24}, ...$

19. Suppose $4^a 6^b = 96$ for integers a and b. What is $a + b$?

20. Prime numbers are numbers that are only divisible by itself and 1. The first few are $2, 3$, and 5. For example, 4 is not prime because it is divisible by itself and 1, but also 2. Suppose two prime numbers sum to a multiple of 11. Find the smallest possible value of the smaller of the two primes.

21. In the diagram below, square $ABCD$ has side length 10. A second square $PQRS$ also has side length 10 with point P being the center of $ABCD$. It is rotated such that it intersects square $ABCD$ at two points M and N such that $MC = 3$ and $NC = 7$. What is the area of quadrilateral $PMCN$?



22. Mia is listing the numbers from 1 to 105. Find the number of 1's that she writes down.

23. What is the units digit of 2024^{2024}?

24. Ally rolls a 6-sided die twice and forms a two-digit number whose tens digit is the first number she rolled and whose units digit is the second number she rolled. For example, if she rolled a 3 and a 5, then she forms the number 35. What is the probability, expressed as a common fraction, that the number she forms is divisible by 6?

25. A Googol is equal to 10^{100}, which is 1 followed by 100 zeroes. Find the sum of the digits of a Googol minus 1234.

26. For some prime number n, the base-n numbers 21_n and 16_n are prime. Find the smallest possible value of n.

27. A teacher wants her 8 students to get in a single file line. If four students want to stand next to each other (in no particular order), how many ways can she create this line?

28. A triangular number can be expressed as the sum $1 + 2 + 3 + ... + n$ for some positive integer n, with the first three triangular numbers being 1, 3, and 6. What is the sum of the 7th and 8th triangular numbers?

29. Let a, b, c, d be positive integers that satisfy $a + \frac{1}{b + \frac{1}{c + \frac{1}{d}}} = \frac{425}{123}$. Find $a + b + c + d$.

30. A standard deck of 52 cards is shuffled and two cards are selected from it. What is the probability, expressed as a common fraction, that the first card is an Ace and the second is a Spade? A standard deck of cards has 4 suits, each with one of the 13 types of cards.